

## VISION BASED ALTITUDE CONTROL FOR A TRAJECTORY FOLLOWING QUADROTOR USING POSITION FEEDBACK

Bharath.M.K<sup>1</sup>, Hemanth Kumar<sup>2</sup>, Ashish Prasad<sup>3</sup>, Gokul.M<sup>4</sup> and S.M.Vaitheeswaran<sup>5</sup>

<sup>1,2</sup> *Project Assistant, NAL, CSIR, Bangalore, Karnataka, 560017*

<sup>3</sup> *Student (M.Tech, NITK Surathkal), Mangalore, Karnataka, 575025*

<sup>4</sup> *Student (M.Tech, Amrita Vishwa Vidyapeetham), Coimbatore, TamilNadu, 641112*

<sup>5</sup> *Principal Scientist, ALD, NAL, CSIR, Bangalore, Karnataka, 560017 (corresponding author)*

### Abstract

A quadrotor, or quadrotor helicopter, is an aircraft that becomes airborne due to the lift force provided by four rotors usually mounted in cross configuration, hence its name. They are of particular interest due to their small size, great maneuverability and hovering capability. However quadrotors have a limited payload capacity because they require four motors for operation, which consumes more power than conventional Unmanned Aerial Vehicles. This reduces the number of sensors and mission critical equipment they are able to carry. Due to this limitation researchers have turned to vision-based techniques for navigation of UAVs. Vision-based navigation provides a large number of useful features that extend quadrotors capabilities. They include the basic control functions of keeping the quadrotor roll and pitch angles stable (attitude control), controlling the vehicle trajectories when flying in free spaces (course stabilization), and keeping the altitude at proper height over ground. A color camera tracks the image features under the quadrotor and above ground. These blobs are located on a known geometric shape.

### Nomenclature

YPR	=	Yaw, Pitch, Roll
PID	=	Proportional Integral Derivative
UAV	=	Unmanned Aerial Vehicle
$\theta$	=	Roll angle
$\varphi$	=	Pitch angle
$\psi$	=	Yaw angle
$v$	=	Speed vector
$R$	=	Rotation matrix
$F_i$	=	Forces due to $i$ rotor
$C$	=	Force to moment scaling factor
$u_1$	=	Total thrust on the body in z-axis
$g$	=	Acceleration due to gravity
$u_2, u_3, u_4$	=	Pitch, roll and yaw moment

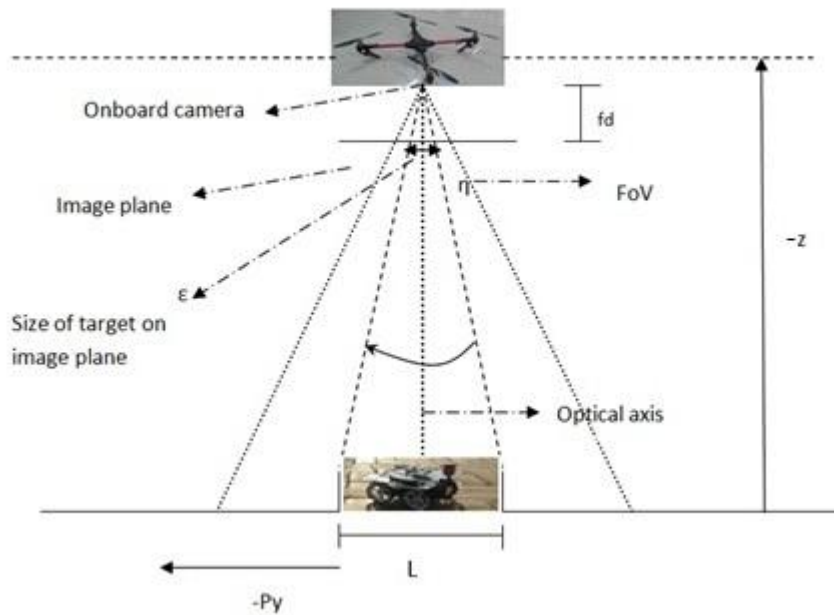
## I. Introduction

A quadrotor also known as quadrotor unmanned aerial vehicle (UAV) consists of two pairs of counter-rotating rotors and propellers, located at the vertices of a square frame. It has four rotors and is capable of vertical take-off and landing (VTOL), and does not require complex mechanical linkages, such as swash plates or teeter hinges that commonly appear in typical helicopters. Due to its simple mechanical structure, it has been envisaged for various applications that include terrain and utilities inspection, disaster monitoring, environmental surveillance, search and rescue, law enforcement and traffic surveillance, agriculture, indoor applications etc. The quadrotor's movement is controlled by varying the relative thrusts of each rotor. While traditional helicopters generate most of their lift with a single blade, the weight of a quadrotor is split between

four separate rotors, two of which rotate opposite the other two. Because each rotor generates a portion of the total lift, the flight of a quadrotor is controlled by varying the relative lift and torque of each rotor.

For example, to roll or pitch, one rotor's thrust is decreased and the opposite rotor's thrust is increased by the same amount. This causes the quadrotor to tilt. When the quadrotor tilts, the force vector is split into a horizontal component and a vertical component. This causes two things to happen: First, the quadrotor begins to travel opposite the direction of the newly created horizontal component. Second, because the force vector has been split, the vertical component will be smaller, causing the quadrotor to begin to fall. In order to keep the quadrotor from falling, the thrust of each rotor must then be increased to compensate. This is done using a vision system in this paper.

A quadrotor is equipped with a fixed vision camera looking ahead. This camera captures the image of a pattern mounted on the target vehicle that features four marks on a square of known side length. It is assumed that the height of the horizontal median of this square coincides with the height of the image's camera center. The control aim is, the quadrotor follows a target vehicle evolving with unknown motion on the working area. The positions of the pattern's marks on the image are expressed in pixels. From these image features, as measured by the vision system, it is possible to compute the posture of the target vehicle, measured on a coordinate system associated to the camera. Figure 1. shows a diagram with the horizontal projection of the vision system, showing the posture of the target vehicle on the camera's coordinates [1].



**Figure 1: Front-view of the scenario for controlling altitude based on camera tracking.**

## II. System Architecture

Dynamical model of the four-rotor quadrotor is developed in simulation in which a quadrotor aerial vehicle is visual-servo controlled. The proposed system's block diagram is presented in Figure 2. The block diagram consists of a vehicle block which incorporates the vehicle dynamics, camera model, sensors and filtering stage. The control Module and estimation Module, use the Kalman filtering theory for state observation, and the reference trajectory model as a complement for setting accuracy attitude references.

The control design proceeds by developing PID control equations in order to control: target position tracking and altitude.

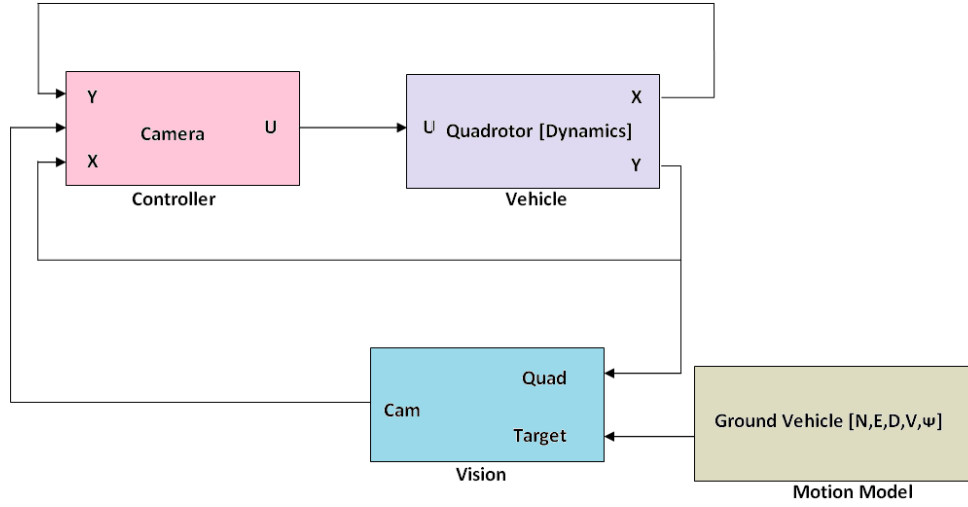


Figure 2: Block Diagram of Control and Navigation Using Vision Aided Altitude Control.

### III. Quadrotor Dynamics and Control

A body fixed frame is assumed to be at center of gravity of the quadrotor, where the z-axis is pointing upwards. This body axis is related to the inertial frame by a position vector  $(x,y,z)$  and 3 Euler angles,  $(\theta, \phi, \psi)$ , representing pitch, roll and yaw respectively. A ZYX-Euler angle representation given in Equation (1) has been chosen for the representation of the rotations [2].

$$R = \begin{pmatrix} c_\phi c_\theta & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ s_\phi c_\theta & s_\phi s_\theta s_\psi - c_\phi c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{pmatrix} \quad (1)$$

Where  $c_\theta$  and  $s_\theta$  represent  $\cos \theta$  and  $\sin \theta$  respectively.

Figure 3 shows the 3D Quadrotor Model, Each rotor produces moments as well as vertical forces. There are four input forces and six output states  $(x,y,z,\theta,\phi,\psi)$  therefore the quadrotor is an under actuated system. The rotation direction of two of the rotors are clockwise while the other two are counterclockwise, in order to balance the moments and produce yaw motion as needed [2],[3].

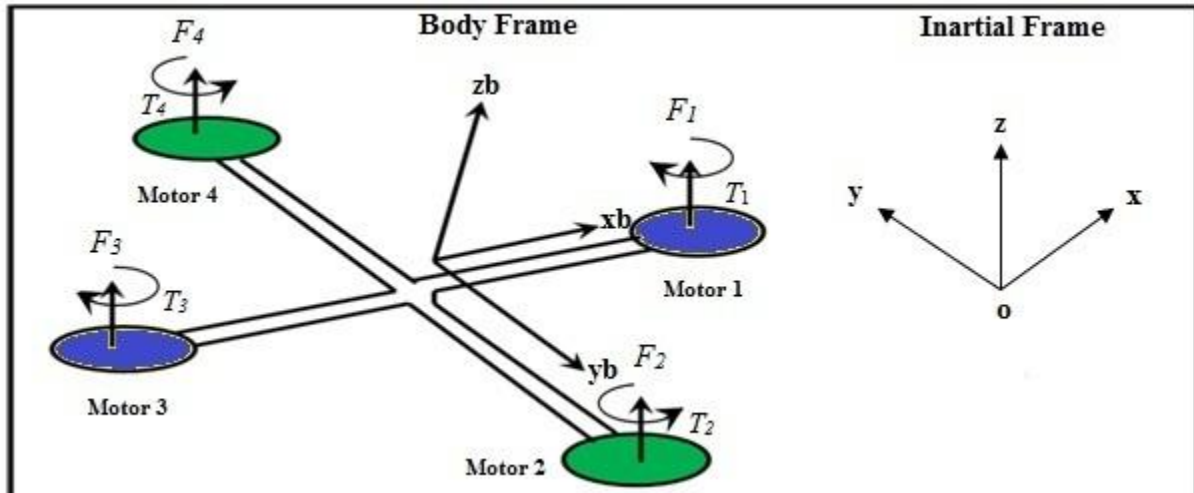


Figure 3: 3D Quadrotor Model

The equation of the motion can be written using the force and moment balance.

$$\begin{aligned}
 \ddot{x} &= \frac{\left(\sum_{i=1}^4 F_i\right)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) - K_1 \dot{x}}{m} \\
 \ddot{y} &= \frac{\left(\sum_{i=1}^4 F_i\right)(\sin \phi \sin \theta \cos \psi + \cos \phi \sin \psi) - K_2 \dot{y}}{m} \\
 \ddot{z} &= \frac{\left(\sum_{i=1}^4 F_i\right)(\cos \phi \cos \psi) - mg - K_3 \dot{z}}{m} \\
 \ddot{\theta} &= l \left( -F_1 - F_2 + F_3 + F_4 - K_4 \dot{\theta} \right) / J_1 \\
 \ddot{\psi} &= l \left( -F_1 + F_2 + F_3 - F_4 - K_5 \dot{\psi} \right) / J_2 \\
 \ddot{\phi} &= l \left( M_1 - M_2 + M_3 - M_4 - K_6 \dot{\phi} \right) / J_3
 \end{aligned} \tag{2}$$

The  $K_i$ 's given above are the drag coefficients. In the following we assume the drag is zero, since drag is negligible at low speeds. For convenience, we will define the inputs to be

$$\begin{aligned}
 u_1 &= (F_1 + F_2 + F_3 + F_4) / m \\
 u_2 &= (-F_1 - F_2 + F_3 + F_4) / J_1 \\
 u_3 &= (-F_1 + F_2 + F_3 - F_4) / J_2 \\
 u_4 &= C(F_1 - F_2 + F_3 - F_4) / J_3
 \end{aligned} \tag{3}$$

Where  $J_i$ 's are the moment of inertia with respect to the axes and  $C$  is the force-to-moment scaling factor. The  $u_1$  represents a total thrust on the body in the z-axis,  $u_2$  and  $u_3$  are the pitch and roll inputs and  $u_4$  is a yawing moment. Therefore the equations of motion become

$$\begin{aligned}
 \ddot{x} &= u_1 (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\
 \ddot{y} &= u_1 (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\
 \ddot{z} &= u_1 (\cos \theta \cos \psi) - g \\
 \ddot{\theta} &= u_2 l \\
 \ddot{\psi} &= u_3 l \\
 \ddot{\phi} &= u_4
 \end{aligned} \tag{4}$$

The control strategies used to command the Quadrotor while tracking the target on ground. The control design proceeds by developing PID control equation in order to control target position tracking and altitude while tracking the target with an embedded camera, is shown in Figure 1.

$$\ddot{z} = u_z = -g + c\phi\theta \frac{U_1}{m} \tag{5}$$

#### IV. Camera Modeling

The camera model (shown in Figure.1) is used for tracking the target on ground in order to control altitude and forward/backward motion. The position of the target is given by the vector  $P_{[xyz]}$ , which corresponds to the pixel location on image plane:  $(\varepsilon_x, \varepsilon_y)$ . The objective is to project the target position onto the frame of reference of the camera. These relationship is obtained by relating the camera field-of-view (FoV): $\eta$ , the height above ground  $-z$ , the lateral position error  $P_y$ , the roll angle  $\phi$ , and the total number of pixels along the lateral axis of the camera:  $M_x$  and  $M_y$ . The control objective is to maintain the vehicle in a constant altitude ( $z$ ), while tracing the trajectory defined by the target on ground [3][4][5].

To establish an equation the real size of the target ( $L$ ) is related to its size in the image plane  $\varepsilon$  (given in pixels), the focal distance  $f_d$  and the altitude to hold  $z$ . This equation is easily found using simple triangles relation as:

$$\frac{-z}{L} = \frac{f_d}{\varepsilon} \quad (6)$$

$$\ddot{z} = u_z = \frac{f_d}{L} \frac{\ddot{\varepsilon}}{\varepsilon^2} - 2f_d L \frac{\dot{\varepsilon}}{\varepsilon^3}, \quad (7)$$

$$\ddot{\varepsilon} = u_z \left( \frac{\varepsilon^2}{f_d L} \right) + 2 \frac{\dot{\varepsilon}}{\varepsilon^3} \quad (8)$$

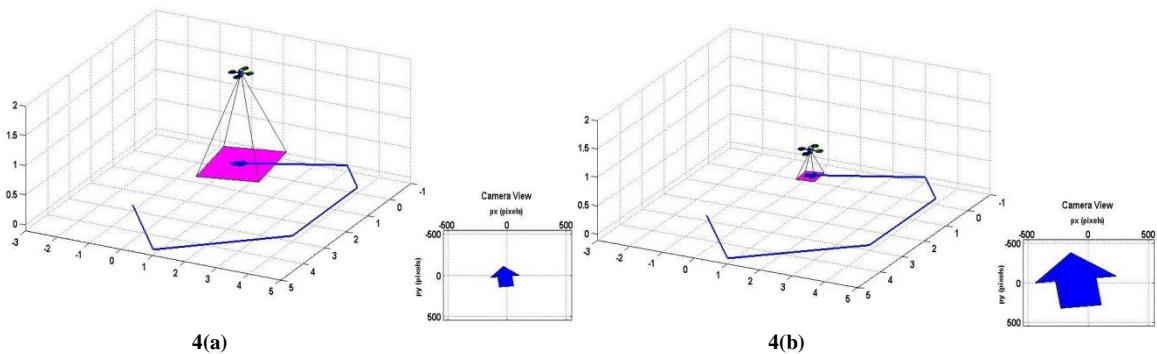
$$PID = K_{p,z} (\varepsilon^d - \varepsilon) - K_{d,z} \dot{\varepsilon} + K_{i,z} \int (\varepsilon^d - \varepsilon) d\tau \quad (9)$$

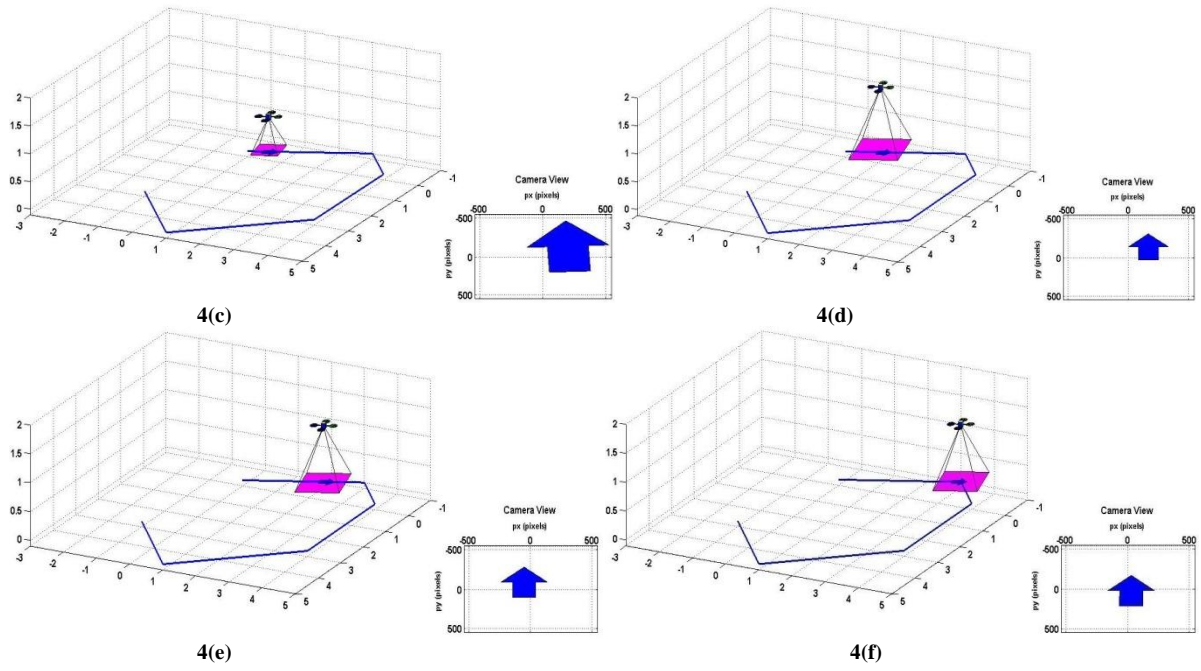
Where  $K_{p,z}$ ,  $K_{d,z}$  and  $K_{i,z}$  are the proportional, differential and integral gains respectively

$$u_z = \frac{f_d L}{\varepsilon^2} K_{p,z} (\varepsilon^d - \varepsilon) - \left( \frac{f_d L}{\varepsilon^2} K_{d,z} + 2 \frac{f_d L}{\varepsilon} \right) \dot{\varepsilon} + \frac{f_d L}{\varepsilon^2} K_{i,z} \int (\varepsilon^d - \varepsilon) d\tau \quad (10)$$

#### V. Experimental Results

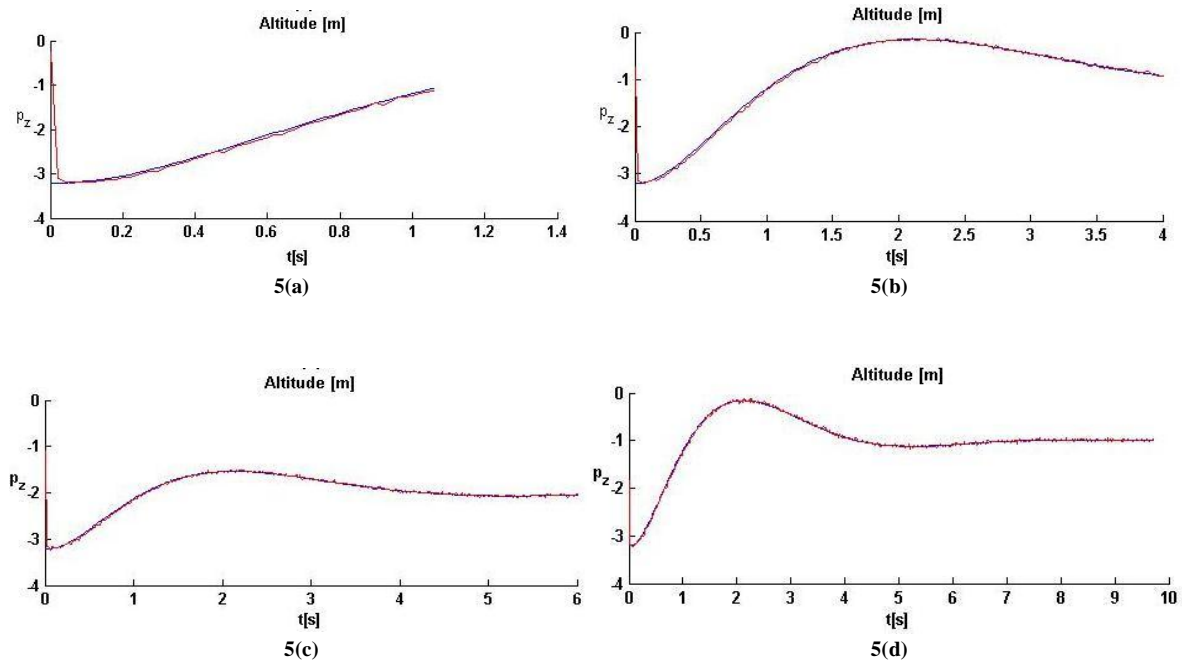
A Matlab simulation of quadrotor altitude stabilization is implemented. The quadrotor pose is estimated using a blob estimation algorithm. The algorithm uses a 2.5 cm radius colored blobs that are attached to the ground vehicle. A blob tracking algorithm is used to get the positions and areas on the image plane. Equation (6) is used for the height estimation. Equations (7) through (9) are used to estimate using set point control from the images processed on ground. The rotor speeds are set according to achieve the desired positions and orientations. For the experiment, the quadrotor is made to maintain at a height of 1m.. The PID controller estimates the error in the altitude and gradually reduces the error by repeated number iterations. Figure 4. Shows a completely stabilized portion of the quadrotor where the minimum amount of thrust is applied to keep the quadrotor at the given height.

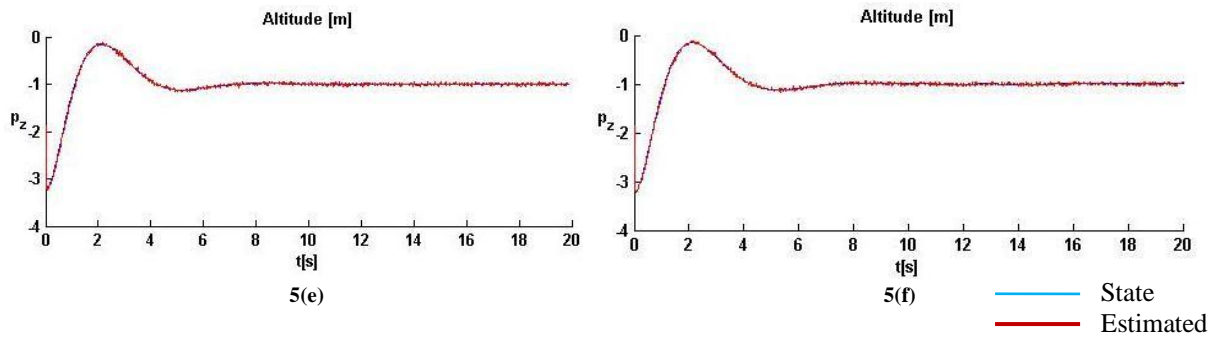




**Figure 4: Simulation results for vision based Altitude control of quadrotor.**

Figure 4(a) to Figure 4(f) shows the quadrotor Altitude stabilization process, where figure 4(a) shows the initial altitude state of the quadrotor. In this state the quadrotor goes above set altitude value. Figure 4(b) shows the error calculation of altitude from the set altitude and based on the error difference quadrotor initiates to stabilize. In this process the quadrotor goes down the set altitude which is shown in figure 4(b). Figure 4(c) shows the further estimation of error in altitude from the set altitude and based on this error information the quadrotor gets further stabilized. Figure 4(d) shows the quadrotor stabilized to the set altitude. Figure 4(e) and Figure 4(f) shows the altitude of the quadrotor remains in steady stabilized state as target moves along the set trajectory. The blue arrow indicates the camera view of the target that indicates the Target size and direction as the quadrotor stabilizes.





**Figure 5: Simulation results for Vision based Altitude stabilization**

The experimental results shows the variation of altitude and its stabilization to the set altitude, In the simulation results Figure 5(a), to Figure 5(d), shows the Transient response of altitude control. Figure 5(e) and Figure 5(f) shows Stable steady state altitude of quadrotor to the set altitude.

The proposed vision-based estimation and control strategies were implemented in real time with the quadrotor as the vision based follower and the ground vehicle being the autonomous leader. Experiments were performed for hovering over a stationary target based on vision-sensing alone and for tracking of a moving target.

## VI. Conclusion

Design and implementation of a controller for a quadrotor were investigated. The primary focus was on a task of a vision-based target tracking. The goal of the work was to complete implementation of a system capable of tracking a moving target based on limited information about the target and vision-sensing alone. Proportional-Integral-Derivative control was used for stabilization of altitude of a quadrotor.

## Acknowledgments

The authors SMV, MKB and HKE, NAL gratefully acknowledge the funding received from AR&DB, GoI under the NPMICAV program to carry out the above work. The work is carried out at National Aerospace Laboratories, CSIR, Bengaluru, Karnataka, INDIA.

## VII. References.

- [1] A. Barrientos, and J. Colorado, Miniature Quad-rotor Dynamics Modeling & Guidance for Vision-based Target Tracking Control Tasks. in IEEE International Conference, May 11, 2009.
- [2] Erdinc Altug, James P. Ostrowski, Robert Mahony, "Control of a quadrotor Helicopter Using Visual Feedback," in IEEE International Conference on Robotics and Automation, May. 2002.
- [3] Luukkonen, Teppo. "*Modelling and control of quadcopter*". Aalto University, School of Science. August 22, 2011
- [4] E. Altug, J. P. Ostrowski, and C. J. Taylor, "Quadrotor Control Using Dual Camera Visual Feedback" in IEEE International Conference on Robotics and Automation, 2003. 10, 89.
- [5] T. Pornsin-Shiriak, Y. Tai, H. Nassef, and C. Ho, "Titanium-alloy MEMS wing technology for a micro aerial vehicle application," J. of Sensors and Actuators A: Physical, vol. 89, pp. 95–103, Mar. 2001.